Input and Output Power Balance in Finite-Element Analysis of Electric Machines Taking Account of Hysteretic Property

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This paper investigates the instantaneous power balance of three phase rotating machine in the finite element magnetic field analyses. Moreover, we discuss the influence of hysteretic properties of magnetic materials to the several instantaneous powers of an interior permanent magnet synchronous motor in the finite element magnetic field analyses using the isotropic vector play model.

***Index Terms*— Electromagnetic fields, finite element methods, magnetic hysteresis.**

I. Introduction

IN order to accurately estimate the iron loss of electric machines, it is necessary to consider hysteretic property of magnetic materials in magnetic field analyses. For example, the Preisach model [1], [2] and its extended versions have been widely used to represent hysteretic characteristics.

In general, the iron loss is calculated in post-processing based on the magnetic flux density distribution obtained from magnetic field analyses by using a magnetization curve [3]. Reference [4] has been developed a method for post-correction of voltage/current and electromagnetic force obtained from magnetic field analyses by using a magnetization curve. In this paper, we investigate the instantaneous power balance and the influence of the hysteretic property by using the play model [5], [6], which is mathematically equivalent to the Preisach model, to the several instantaneous powers of an interior permanent magnet synchronous motor in a finite element magnetic field analysis.

II. Property of Play Model

A discretized form of the isotropic vector play model [5] can represents the hysteretic properties with an output of magnetic field $\boldsymbol{h}$ from an input of magnetic flux density $\boldsymbol{b}$ because the finite element analysis using magnetic vector potential as an unknown variables requires the calculation of $\boldsymbol{h}$ from $\boldsymbol{b}$.

$$\boldsymbol{p}_{\zeta k}(\boldsymbol{b}) = \boldsymbol{b} - \frac{\boldsymbol{b} - \boldsymbol{p}_{\zeta k}^{*}}{\max\left(\left|\boldsymbol{b} - \boldsymbol{p}_{\zeta k}^{*}\right|, \zeta_k\right)}\zeta_k , \tag{1}$$

$$\boldsymbol{h} = \sum_{k=0}^{M-1} f_{\zeta_k}\left(\left|\boldsymbol{p}_{\zeta_k}(\boldsymbol{b})\right|\right)\frac{\boldsymbol{p}_{\zeta_k}(\boldsymbol{b})}{\left|\boldsymbol{p}_{\zeta_k}(\boldsymbol{b})\right|}, \tag{2}$$

where $\boldsymbol{p}_{\zeta k}$ is the play hysteron operator with a width of ζ_k, $\zeta_k = kB_{\max}/M$, $B_{\max}$ is the maximum measurable magnetic flux density, $\boldsymbol{p}_{\zeta k}^{*}$ is the play hysteron operator at previous time step, M is the number of the play hysterons, and $f_{\zeta k}$ is the shape function for the play hysteron operator $|\boldsymbol{p}_{\zeta k}|$. The identification method for the Preisach model [1], [2] can be applied to the play model because the play model is mathematically equivalent to the Preisach model.

III. Finite Element Magnetic Field Analysis

A. Instantaneous Power Balance

The instantaneous power balance of three phase (U-phase, V-pahse, and W-phase) rotating machine can be written as

$$p_{\text{in}}(t) = p_{\text{out}}(t) + w_{\text{iron}}(t) + w_{\text{cu}}(t) + w_{\text{mag}}(t), \tag{3}$$

$$p_{\text{in}}(t) = v_{\text{U}}(t)i_{\text{U}}(t) + v_{\text{V}}(t)i_{\text{V}}(t) + v_{\text{W}}(t)i_{\text{W}}(t), \tag{4}$$

$$p_{\text{out}}(t) = \omega T(t), \tag{5}$$

$$w_{\text{iron}}(t) = \iiint_{V_{\text{iron}}} \boldsymbol{h}(t)\cdot\frac{\mathrm{d}\boldsymbol{b}(t)}{\mathrm{d}t}\mathrm{d}V, \tag{6}$$

$$w_{\text{cu}}(t) = R\left\{i_{\text{U}}^{2}(t) + i_{\text{V}}^{2}(t) + i_{\text{W}}^{2}(t)\right\}, \tag{7}$$

$$w_{\text{mag}}(t) = \iiint_{V_{\text{noniron}}} \boldsymbol{h}(t)\cdot\frac{\mathrm{d}\boldsymbol{b}(t)}{\mathrm{d}t}\mathrm{d}V, \tag{8}$$

where p_{in}, p_{out}, w_{iron}, w_{cu}, and w_{mag} respectively indicate input power, output power, magnetic energy in the iron core, copper loss, magnetic energy excluding an iron core, and v, i, ω, T, R, V_{iron}, and V_{noniron} respectively indicate voltage, current, angular velocity, torque, phase resistance, volume of the iron core, and volume of the analyzed model excluding the iron core such as the air region and magnet. A discretized forms of (3) – (8) can be written as

$$p_{\text{in}}{}^{n} = p_{\text{out}}{}^{n} + w_{\text{iron}}{}^{n} + w_{\text{cu}}{}^{n} + w_{\text{mag}}{}^{n}, \tag{9}$$

$$p_{\text{in}}{}^{n} = v_{\text{U}}{}^{n}i_{\text{U}}{}^{n} + v_{\text{V}}{}^{n}i_{\text{V}}{}^{n} + v_{\text{W}}{}^{n}i_{\text{W}}{}^{n}, \tag{10}$$

$$p_{\text{out}}{}^{n} = \omega T^{n}, \tag{11}$$

$$w_{\text{iron}}{}^{n} = \sum_{i}^{N_{\text{iron}}} \frac{V_i}{2\Delta t}(\boldsymbol{h}_i{}^{n} + \boldsymbol{h}_i{}^{n-1})\cdot(\boldsymbol{b}_i{}^{n} - \boldsymbol{b}_i{}^{n-1}), \tag{12}$$

$$w_{\text{cu}}{}^{n} = R\left\{i_{\text{U}}{}^{n2}(t) + i_{\text{V}}{}^{n2}(t) + i_{\text{W}}{}^{n2}(t)\right\}, \tag{13}$$

$$w_{\text{mag}}{}^{n} = \sum_{i}^{N_{\text{noniron}}} \frac{V_i}{2\Delta t}(\boldsymbol{h}_i{}^{n} + \boldsymbol{h}_i{}^{n-1})\cdot(\boldsymbol{b}_i{}^{n} - \boldsymbol{b}_i{}^{n-1}), \tag{14}$$

where n, N_{iron}, and N_{noniron} respectively indicate time step, the number of elements in the iron core, and those in analyzed model excluding the iron core.

B. Numerical Results

Fig. 1 and Table I show the analyzed model and condition of an interior permanent magnet synchronous motor (IPMSM). The number of elements and nodes are 7,644 and 3,871, respectively. The play model is identified from 40 symmetric loops of the non-oriented electrical steel sheet JIS: 50A470 at intervals of 0.05 T from 0.05 T to 2.0 T.

Fig. 2 shows the numerical results of the several instantaneous powers obtained from the finite element magnetic field analysis with a magnetization curve and hysteretic property by using the play model. Almost the same higher harmonics of the several powers are obtained regardless of whether hysteretic property is considered or not. However, considering the hysteretic property in finite element magnetic field analysis, the average input power per period slightly increases and the average output power per period decreases because of the influence of the iron loss. Therefore, a magnetic field analysis taking account of hysteretic properties influences not only decreasing the output power but also increasing the input power. The further numerical results will be reported in the full paper.

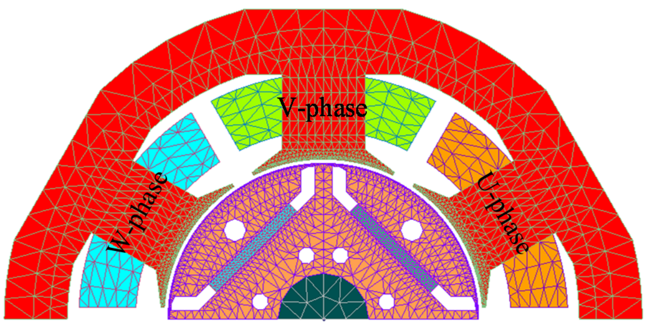
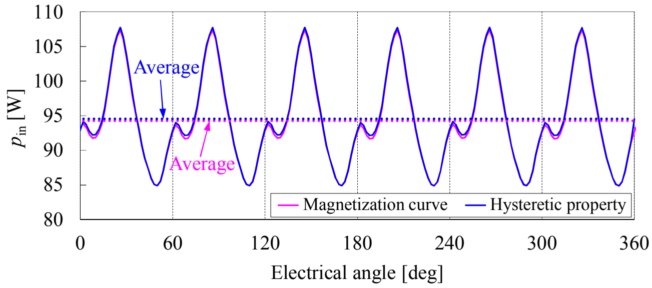


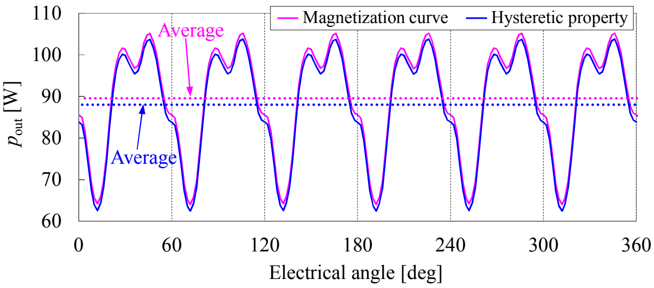
Fig. 1. Analyzed mesh for IPMSM.

TABLE I
ANALYSIS CONDITION

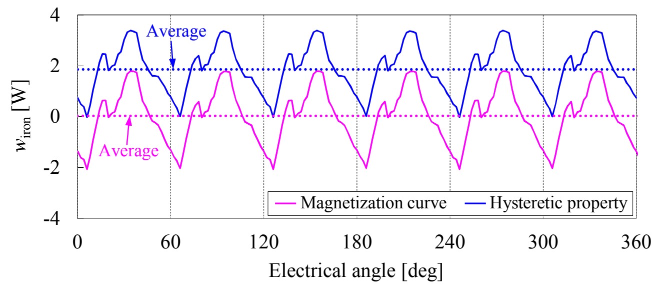
Outer diameter of stator core [mm]	102.2
Outer diameter of rotor core [mm]	54.6
Magnet size [mm]	20.0 × 1.98
Air gap [mm]	0.7
Core length [mm]	60
Number of turns [turns/teeth]	125
Residual magnetic flux density [T]	1.225
Residual relative permibility	1.05
Iron core of stator and rotor	JIS: 50A470
Phase resistance [Ω]	0.38
Revolutionary speed [r/min]	1,500
Number of time steps per period	180
Amplitude of current waveforme [A]	2
Phase angle of current waveforme [deg]	20



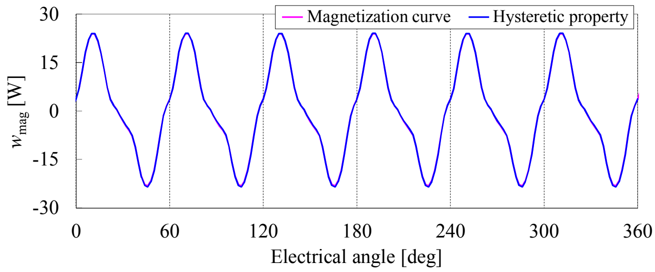
(a) Input power p_{in}



(b) Output power p_{out}



(c) Magnetic energy in iron core w_{iron}



(d) Magnetic energy exculding iron core w_{mag}

Fig. 2. Comparison between several instantaneous powers.

REFERENCES

[1] I. D. Mayergoyz, *Mathematical Models of Hysteresis and Their Applications*, Spring-Verlag, New York (2003).

[2] E. D. Torre, *Magnetic hysteresis*, Picataway, NJ:IEEE Press (1999).

[3] K. Yamazaki and N. Fukushima, "Torque and Loss Calculation of Rotating Machines Considering Laminated Cores Using Post 1-D Analysis," *IEEE Trans. Magn.*, vol. 47, no. 5, pp. 994-997 (2011).

[4] M. Sakashita, K. Nishi, S. Ito, T. Mifune, and T. Matsuo, "Post-Correction of Current/Voltage and Electromagnetic Force for Efficient Hysteretic Magnetic Field Analysis," *IEEE Trans. Magn.* (submitted).

[5] T. Matsuo and M. Shimasaki, "Two Types of Isotropic Vector Play Models and Their Rotational Hysteresis Losses," *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 898-901 (2008).

[6] J. Kitao, K. Hashimoto, Y. Takahashi, K. Fujiwara, Y. Ishihara, A. Ahagon, and T. Matsuo, "Magnetic Field Analysis of Ring Core Taking Account of Hysteretic Property Using Play Model," *IEEE Trans. Magn.*, vol. 48, no. 11, pp. 3375-3378 (2012).